Risk profile assessment embedded into the Bayesian framework

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Abstract. Adverse events in organizations are more than a serious concern. Over the last few years the awareness of this problem has raised and different organizational solutions have been tried. We focus on the problem of managing operational and clinical risks, in terms of events that influence the success of service delivery. This paper is aimed at proposing risk management as the basic methodological approach to deal with adverse events and risks. We propose Bayesian networks (BNs) to assess risk profiles given a context of application and benchmarks by Bayesian decisional theory to evaluate the profiles, i.e. defining the acceptability of them.

The method is described both at a theoretical and an empirical level, thanks to its application to health care (haemodialysis department) and banking field. The occurrences of these top events are modeled by Bayesian networks which gather posterior risk profiles for each patient or banking business line. The comparison of them with a reference risk profile is input for decision making. BNs augmented with decisional nodes and scenario analysis complete the risk management process. The ultimate goal is to improve risk profile and, consequently, service supply quality in the organization.

Keywords: Bayesian network, Distance measure, Bayesian decision theory, Risk management, Risk assessment, Predictive risk profile, Operational risk, Clinical risk

1 Introduction

Nowadays information technology monitoring systems collect large masses of data by close time windows (i.e. hours, minutes, seconds). Each application context needs of informations on which risk managers periodically assess risks. The dynamic nature of risks and the lack of some key informations require to extract all available knowledges through the merging of different data sources, e.g. coming from consortia, self–assessment of experts and internal collection. A risk is defined as a set of scenarios, each of which combines uncertainty and exposure, quantified by probability of the frequency with which an event might take place (uncertainty about the frequency) and by probability of the severity of the consequence (uncertainty about the severity of the consequence).

Among several risk categories this paper focuses on operational and clinical risk. Operational risk (OR) involves all activities with a poor organizational control environment and the term stresses the role of pertaining to the delivery of services [1, Alexander (2003)]; health care organizations (HCOs) emphasize the role of the particular service supplied, i.e. the functioning of the facility, the health care delivery focusing on patient safety, and introduce the clinical risk term (CR) [10, Lorton (2005)].

Internal and external industrial environments, such as new activities or machineries, service supply processes, markets or customer (also patients) modify quickly risk business profiles; the risk manager has to forecast adverse events and to detect the unacceptable risks. Furthermore, to forecast possible harmful events and to activate control systems the data have to be update on horizon time.

In the present work we intend a methodology for the evaluation of operational and clinical risk profiles, useful for different business domains. The method will be described both at the theoretical level and empirical one, involving health care and banking fields as examples.

This paper is structured as follows: in Section 2 we introduce some technical concepts on risk management process and its phases embedded into a Bayesian framework; in Section 3 we propose Bayesian Networks (BNs) to assess risks, measures of distance, and Bayesian decision theory to evaluate posterior risk profiles. In Section 4 we present the application of the methods to assess the risk level of single patients through available data; finally Section 5 summarizes the most important results.

2 Risk management process and risk profile assessment

Risk management is the reduction of the frequency and/or of the impact of a negative event causing damage and adverse effects. International and national organizations propose many guidelines to realize a correct and efficient risk management. Among them, the framework of our research is based on the recommendations and the technical reports of the Italian Healthcare Department "Ministero della Salute" [5, Commissione Tecnica sul Rischio Clinico (2004)] and of the "Basel Committee on Banking Supervision" [3, (2004)].

Risk management is the process of measuring, or assessing risk and developing strategies to manage it. Strategies include transferring the risk to another party, avoiding the risk, reducing the negative effect of the risk, and accepting some or all of the consequences of a particular risk. The above elements compound the risk management process. In particular, it comprehends two aspects: *assessment* and *control*. On its turn, clinical and operational risk assessment involves *risk analysis* and *risk evaluation* (risk aversion, for our purpose). The former is the systematic use of available data sources to identify OR and CR and estimate them; risk analysis includes the choice of a methodology given the context (e.g. HCO, banking, department, business line, customer or patient); the latter is the judgment, on the basis of risk analysis and environment, of whether a risk is acceptable (for example, through cost/benefit analysis or outcome results versus targets). Whenever a risk is not acceptable, it is necessary to take decisions, in order to avoid it or to reduce its probability. This phase is defined risk control. Risk measurement is the risk management kernel because all practical applications, such as actions, limits and benchmarks, depend on these estimations.

The previous key concepts can be employed to assess risk profiles for the patients and the BL and to estimate an overall risk assessment of the clinical centre or bank; these risk profiles (i.e. the most critical failures for clinical and operational target) will allow to define decision support strategies [6, Cornalba C. (2006)] able to minimize risks.

Our objectives may be realized using different models, each of them with advantages and drawbacks. *Generalized survival models* [11, McCulloch et al. (2001)] are not adequate for catching CRs and ORs, although they are approaches applied in biostatistics to estimate relative risk (RR). They lack of severity of impact, a necessary condition to estimate risks. Probability theory and graph theory merge into *graphical models* which are a natural tool for dealing with uncertainty and complexity of risks. Many prior knowledge representations are available, such as *decision tree* and *artificial neural network*, *rule bases–model*, and *Bayesian network*. Thanks to its modularity, a graph based model splits complex system into simpler combined parts depicting the domain and the specific goals (more then one is possible). Neural networks and Bayesian networks are graphically more similar then decision trees, although neural networks do not organize decision problems in a convenient way. A decision tree picks out causal effect relationships between inputs (explanatory variables) and outcome, but not among inputs.

Bayesian theory uses all the available information, even if that information comes from sources outside the experiment. The Bayesian posterior depends on data only through the likelihood, which is calculated from real data (i.e. actually observed). Bayesian methods are often recommended as the proper way to make formal use of subjective information such as expert opinion (physicians or banking experts), prior knowledge from the literature and organization judgments (guidelines and recommendations) or beliefs of an expert.

Because of the above drawbacks we choose the Bayesian approach. Risk analysis may employ Bayesian methods in three ways.

- They are merely tools to *select or parameterize input distributions* for a risk model. The risk model type and the overarching decision process are developed without appeal to Bayesian methods.
  They are applied to *estimate risk distributions*. This use solves a part of the risk management process: the decision aspects are not dealt by the Bayesian analysis.
- They are used as a complete system for *inference and decision making*, taking in count that among rational and coherent decisions the best one to solve a problem belongs to set of Bayesian decisions [14, Wald (1950)]. The Bayesian approach is used to realize the whole risk management process.

In the present research we express and solve clinical scientific and business governance purposes within a full Bayesian framework, as in the third described before. Among the possible Bayesian methods we employ Bayesian Networks [9, Jensen (2001)], hence they are particularly suited for a large number of target variables and introducing of decision theory.

3 Bayesian networks learning from clinical and operational data streams

The Bayesian approach is functional in complex modelling situations where a frequentist analysis is difficult to implement or does not exist; target variables (i.e. top events, like mortality or external fraud) usually have got sparse data in comparison with explicative variables. Bayes' Theorem formalizes the merging of different knowledges, to solve the previous issue.

Let $\mathbf{z} = (x_1, \ldots, x_n)$ a vector of n observations whose probability distribution $\mathrm{p}(z|\theta)$ depends on the values of k parameters $\vartheta = (\theta_1, \ldots, \theta_k)$. Let $\mathrm{p}(\theta)$ the probability distribution of the parameter θ. Then, given the observed data z, the conditional distribution of θ is:

$$p(\vartheta|\mathbf{z}) - \frac{p(\mathbf{z}, \vartheta)p(\vartheta)}{p(\mathbf{z})}. \tag{1}$$

We can write:

$$p(\mathbf{z}) - E[p(\mathbf{z}|\theta)] - c^{-1} - \begin{cases} \int p(\mathbf{z}, \vartheta)p(\vartheta)d\vartheta & \text{for } \vartheta \text{ continuous} \\ \sum p(\mathbf{z}, \vartheta)p(\vartheta) & \text{for } \vartheta \text{ discrete} \end{cases} \tag{2}$$

where the sum or the integral is taken over the admissible range of θ, and $E(\cdot)$ is the mathematical expectation of $f(\theta)$ with respect to the distribution $p(\theta)$. Thus we may write equation (1) alternatively as

$$p(\vartheta|\mathbf{z}) = cp(z|\theta)p(\theta). \tag{3}$$

The statement of equation (1), or its equivalent equation (3), is usually referred to as Bayes Theorem. In this expression, $\mathrm{p}(\theta)$, is called *prior distribution* of θ, or the distribution of θ a priori and it tells us what is known about θ without knowledge of the data. Correspondingly, $\mathrm{p}(\theta|z)$, is called the *posterior distribution* of θ given z, or the distribution of θ a posteriori and it tells us what is known about θ given knowledge of data, . The quantity c is merely a "normalizing" constant necessary to ensure that the posterior distribution $\mathrm{p}(\theta|z)$ integrates or sums to one.

The main issues to implement empirically risk management process usually are represented by high dimensional distributions, lacking of data and decision aspects. Furthermore, the huge number of variables does not allow to use computational shortcuts; graph models implemented in software programs may be able to solve the problem using computer–intensive methods. BNs are particularly suited to manage poor data of OR and CR target variables; they allow clearly to express the relationships (arcs) between the variables (nodes) and to learn risk profiles from the available data or prior information. Furthermore, they may add knowledge on the dependences among adverse events and their causes (i.e. human factors, phase of the process) and help to minimize risks [7, Cornalba et al. (2004)] to perform risk assessment and the risk evaluation.

Formally, a BN joint probability distribution for a set of variables $\mathbf{X}=\{X_1, \ldots, X_K\}$ is defined by the pair (S, P), where

- S is a network structure that encodes a set of conditional dependences among variables in $\mathbf{X}$; S has to be DAG, Directed Acyclic Graph;
- P is a set of local probability distributions one for each variable.

The compact representation of the joint probability distribution is

$$p(x_1, \ldots, x_n) = \prod_{i=1}^{k} p(x_i|pa_i) \tag{4}$$

where:

- pa_i denotes the parents of node X_i;
- $p(x_i|pa_i)$ denotes the local probability distribution of the variable X_i, conditionally on its parents.

Probabilistically, a variable is independent of its ancestors conditional on its parents; the conditional independence property is known as Markov property. Given its parents and its children, a variable is conditionally independent of all other variables.

To compact our representation $U = p(x_1, \ldots, x_n)$ we may constrain links among variables; the simplest conditional independence relationships encoded in a Bayesian network can be stated as follows: a node is independent of its ancestors given its parents, where the ancestor/parent relationship is with respect to some fixed topological ordering of the nodes [12, Russell et al. (1995)]. The joint probability distribution is calculated using the chain rule and tacking in count only the probability distribution of the set of parents $pa(A_i)$ of A_i.

$$p(U) = \prod_{i} P(A_i|pa(A_i)). \tag{5}$$

Risk analysts make use of learning to specify the graph topology (S) and the parameters of each conditional probability distribution. In particular, the topology may be known or unknown: they may set deterministically causal relationships and then learn new probabilistic relationships from data; on the other hand, they may learn the whole network topology from data. From a statistical point of view, the first approach is computationally more efficient since it restricts the number of possible topologies and it specifies the algorithms that may be implemented (e.g. Maximum Likelihood Estimation, Expectation–Maximization, search through model space or a mix of them); the second one reduces subjectivity fitting topology only on data, but learning structure is much harder than learning parameters, especially whenever there are missing data or hidden nodes. From a medical and banking point of view, the first approach allows to visualize relationships and causes generating an event; the second one is harder to apply mainly for validation difficulty of new relationships.

3.1 Distance measures and benchmarks for posterior risk profiles

Suppose that causality aspects, simplifications of the models through mediating variables and feed back cycles are solved; we have a limited number of structures among which we have to choose the best structure hypothesis. To validate topology and to compare posterior risk profiles some measure of distance may be employed.

Using knowledge $\xi = \{guidelines, literature\}$ and model selection [9, Jensen (2001)] we can minimize the number of possible topologies to submit to experts: this requirement is introduced to avoid situation too specific in which experts are not able to express reasonable estimates concerning the structure hypothesis $\mathrm{P}(\mathrm{S}^h)$.

To establish the credibility of models and select one of them an *acceptance measure* [9] may be applied. The measure takes into account the trade-off between the size of a model and the distance between the "true" distribution and the approximated one by the model. It assesses a score of acceptability which gives a complete order among the hypothesis.

Let M be a Bayesian Network with a set of variables U. For each variable A with parents $pa(A)$ we define $Sp(A)$ to be the number of entries in $P(A|pa(A))$, and the size is

$$Size(M) = \sum_{A \subset U} Sp(A). \tag{6}$$

Let C be the set of configuration over U. Let P denote a "true" distribution over U taken from the sample of database of cases. Let M_i be the i candidate Bayesian Network for P, and let P_i be the joint probability distribution determined by M_i. To compare the two distributions two measures are possible (see Giudici [8, (2003)]):

1. *the Euclidean distance*

$$Dist_E(P, P_i) = \sum_{x \subset U} (P(x) - P_i(x))^2; \tag{7}$$

2. *cross entropy*

$$Dist_C(P, P_i) = -\sum_{x \in U} P(x) \log \frac{P(x)}{P_i(x)}. \tag{8}$$

We have to choose the BN which minimize the acceptance measure:

$$Acc(P, M_i) - Size(M_i) + kDist(P, P_i) \tag{9}$$

where k is a positive real number.

Hence the best topology of BN is selected, the posterior distribution is employed as input in risk analysis to assess the risk profile for each patient, department or business line. In banking and health care domains no benchmarks or reference values about acceptability of risk are available, although they are the start point for every risk assessment and decision on control. Herein, we assume experts and managers evaluate and judge the acceptability of the risk comparing the risk profile of an expected patient (or BL) with that of our patient (or BL). In practice, a reference profile is assessed on prior belief and supposed to be an expected one; the current risk profile for each patient (or BL) is picked out by batch learning. The distance between them evaluates the level of risk for a patient or department.

Let the domain be the health care field and let X_i the i-th variable defining the risk profile for patient j-th. Let be the Euclidean distance measured between the j-th patient distribution and the reference distribution (R):

$$Dist(P_j, P_R) = \sum_{x \in X_i} (P_j(x) \quad P_R(x))^2. \tag{10}$$

The score, defined by the distance, prioritizes the variables on the basis of importance for determining the risk profile. In this way we may clustering key variables by causes. Furthermore, a complete risk evaluation requires beyond to the definition of the standardized acceptability level also a specific acceptability level for each risk on which risk manager decides if the risk has to be controlled for that patient. The specific benchmark depends on own personal belief; for a discussion see Cornalba C. [6, (2006)].

In an uncertain conditions, a risk manager makes decision on the basis of his/her utility function, U(X), i.e. a real valuated function defined on the set of possible consequences of an action. In scaling utilities, for convenience, values are on the unit interval and the function shape depends upon how individual views of risk.

Let X a monetary random variable, U(X) a concave function on an interval (a, b). Let be U(X)=1 the value of utility function with the best possible consequence and U(X)=0 the worst possible one. Let p denote a probability belonging to [0, 1]. Let x_1 and x_2 two points in the interval (a, b), the outcome random variable X=x_1 has probability p. In a gamble, probabilities are interpreted in

terms of betting; the decision maker receives x_1 with probability p, and x_2 with probability $(1-p)$. Let the concavity property of U(X),

$$U[px_1 + (1 - p)x_2] \geq pU(x_1) + (1 - p)U(x_2). \quad (11)$$

The expected monetary value for this gamble is given by E(X)=$[px_1 + (1 - p)x_2]$, lying in the interval (x_1, x_2). If the decision maker takes this gamble, his/her utility increase will be the utility of his/her expected gain, U[E(X)] (for a discussion see Berger [2, (1985)]). For Jensen's Inequality we know that U[E(X)]>E[U(X)], that means the decision maker prefers to select a certain (for sure) increase in money value X=g, compared with taking a gamble that will yield a random gain E(X)=g. Such conservative (or *risk averse*) decision maker perceives a grater utility accepting money with certainty. Analogously, *risk–prone* decision maker has a convex–shaped utility function in the region of a gamble, taking big risk for a large possible gain. *Neutral decision maker* has both concave and convex utility function, i.e. linear shaped. Let θ the *state of nature*, i.e. the parameter defining uncertainty of the decision-making problem. Bayesian rationality is tied to the idea of maximizing expected utility with respect to posterior distributions of θ. Formally, a coherent decision maker has to follow a normative set of axioms developed by Savage [13, (1954)].

Herein we consider only a finite number of possible values taken by θ. For each combination of state of nature and action there will be a consequence. The expected utility is maximized selecting decision by using prior information and available observational data, combined by Bayes' theorem [Eq. 1].

Risk manager employs available information to construct a consistent and coherent set of decisions about the risk assessments and their proper management. In principle, decision maker suffers a loss by taking any decision different from the one that would yield the best possible consequence (zero risk). A loss function may be defined in term of utility function; let $U^B(x)$ the best possible consequence that can occur in a potential gamble, and let $U(x)$ the decision maker's utility for the consequence related to the actual made decision. Let be the loss function a non–negative function express in term of θ, and the action to be taken x

$$L(\theta, x) = regret = \text{ opportunity loss } = U^B(x) - U(x) \geq 0.$$

A risk manager chooses the action that minimizes the expected values of the loss function (with respect to the posterior distribution). Many loss functions have been suggested, such as quadratic, linear and piecewise linear, zero–one, linex asymmetric loss functions; for a discussion see Chib S. et al. [4, (2002)].

4 Applications and experimental results

In this Section we apply our methodology on health care field; in particular, we apply the risk management steps to the haemodialysis (HD) treatment of patients suffering from End Stage Renal Disease (ESRD), a severe chronic condition that corresponds to the final stage of kidney failure. Without medical intervention, ESRD leads to death. The adverse event is the failure to reach a prescribed treatment goal at the end of a dialysis session; this may be due to patient's clinical problems occurring during the dialysis session or to errors of the health care providers; as a consequence, at the end of dialysis sessions, one or more patient monitoring variables will not attain their target.

More than 80% of the ESRD patients are treated with haemodialysis (HD). In HD blood from a patient's body is circulated through an external device or machine and then returned to the patient's bloodstream. The artificial kidney is designed to remove fluids and metabolic end products from the bloodstream; normally, at the end of a dialysis session the acid-base equilibrium is re–established and the water in excess is removed: the device called haemodialysis regulates the overall procedure.

We are interested to measure risk profile of patients and to forecast hospitalization and mortality risks; after the risk assessment phase, we define decisions to improve their therapeutic plans.

4.1 Data stream collection and pre–processing of available data

Data collection coming from health care departments monitor dialysis sessions and operational adverse events. The data warehouse collects two data types. The former type is the data monitored during each patient dialysis session and, in general, HD patients undergo a dialysis session for about four hours three times a week in day-hospital; furthermore, each day OR failures and time to recovery them are collected. The latter comprehends personal data, patients history, and quarterly follow up updates for health care field (such as age, gender, comorbidity, albumin and number of days from the first dialysis session in the department). For a discussion of HD key variables see Cornalba C. [6, (2006)].

In general, the observation vectors are available for each patient over many time points or although one point (at study entry), so the data occupy a three dimensional hyper rectangle – with missing data as well. To manage the "data cube" it has to be represented through flat files with two dimensional data in a relational database.

Among the whole number of available variables, we chose only the variables that affect clinical and OP top events, i.e. mortality, hospitalization and operational risk. In particular, they are analyzed focusing on the importance that have during risk assessment, and causal relationships with others. For example, among 109 clinical variables we select 34 to develop models for HD risk management.

For the purpose, we pre–process data collected solving a few inconsistencies in data: we re-organize our 3 DM array (patients×variables×time) into a HD datamart. Data are summarized through a set of discrete stochastic variables: we transformed continuous into discrete ones following a knowledge based strategy; the number and the length of each interval are defined by research objectives (i.e. operational and clinical risks and associated costs).

Moreover, each dialysis session has been synthesized through the vector of the median values of the monitoring variables; the comparison of the median values with pre defined clinical treatment goals has finally allowed to assess the outcome of the dialysis session, i.e. the presence of an adverse event.

4.2 Bayesian Network and risk profile assessment

Thanks to the availability of prior information we make use of the causal reasoning to limit the number of possible BNs and we consider the arc between two variables as a direct relation of cause effect. Herein we assume the causal network as a graphical representation of causal relationships into the ESRD domain. Exploring the possible parents pa_i of each variable X_i $(i = 1, \ldots, 34)$ we have defined some network topology constraints before the learning phase (EM algorithm), in order to express well–established medical knowledge. Among the available variables, we assume initially being possible only the relationships from a medical point of view. This solution allows to reduce the number (2^{68}) of possible topologies setting constrains (causal, undirected or independence link). In practice, we have defined and limited the space of possible models; then, the scoring function is used to select model belonging to the defined space.

In our case study, we analyze, coeteris paribus, only four topologies due to four variables with reverse link hypothesis. This assumption allows to restrict the number of model M_i with $i = 1, \ldots, 4$. The minimum size is Size(M_3)= 235030 (see Tab. 1). The four models hypothesis have the same size of variables but differing by two possible reverse links between "Haemoglobin" (Hgb) and "Serum Ferritin" (Fe, iron management) and "Serum Calcium" (Ca) and "PTH" (nutritional status). To compare the two distributions we select a symmetric measure in P and P_i. The Euclidean distance satisfies this property. Herein we assume P is a sample from an unknown distribution and not the true distribution; furthermore, we apply an heuristic approach to evaluate P and P_i joint distributions. In principle, we can calculate $P(U)$ as the product of all conditional probabilities from the network. We are interested to employ posterior marginal distribution without being forced to calculate $P(U)$.

Model	Link		Size(M_i)
1	Ca $\mapsto$ PTH	Fe $\mapsto$ Hgb	235034
2	Ca $\mapsto$ PTH	Hgb $\mapsto$ Fe	235102
3	PTH $\mapsto$ Ca	Fe $\mapsto$ Hgb	235030
4	PTH $\mapsto$ Ca	Hgb $\mapsto$ Fe	235098

Table 1. Size of selected models.

Working on the cliques we reconstruct $P(U)$ for the four hypothesis on structures and estimate the component $Dist(P, P_i)$ of the equation 9. The BN minimizing the acceptance measure is again M_3 and we choose it as the best model for ESRD domain, given our knowledge. We remark that here we have not presented in detail the validated BN topology and its key variables; for a discussion see Cornalba C. [6, (2006)].

Our ESRD network topology hypothesis (M_3) may be described through seven "block", where each of them has one (or more than one) top event and base events. In particular, we have:

- IID Department Performances, which gives general performance measures about our outcomes;
- Patient performances, benchmarks defined by nephrologists and localized for our facility;
- Dialysis Quality Indexes, which gives general measures about the quality of the dialysis;
- Resource, which gives general measures on resources utilization;
- Signal variables (they are introduced for construction to improve our knowledge domain on risk); localized for our patients;
- Intermediate Risk Index, the causes which influence the mortality and hospitalization ratio;
  Risk Index: the top events of the research.

Hence the structure is validated, the posterior distribution gathered by BN is employed as input in risk analysis to assess the risk profile for each patient or department. Furthermore, it is possible also the assessment of new risk profile combining knowledge from the different sources and employ reference risk profile as a patient benchmark. This is useful when patient is at the start of the study.

In our case, every three month new data set $y_1, \ldots, y_m$ is used as a prior to update new belief about θ. The new posterior distribution is based upon an equivalent set of (n+m) observations. The effect of earlier experience is to base posterior belief on the totally of data of 6 months, and to increase the sample size. Updating is a necessary phase of risk assessment and control: the risk profile of each patient is evaluated with incoming data and the balance between two sequential risk profiles allows to control the correct implementation of decisions.

To evaluate the risk profile acceptability, we compare the reference posterior distribution with once learned by patient's data. In Table 2 we can see the posterior distributions defining mortality

	Marginal posterior distribution in %			Decision
Variable State	Reference Profile	Mortality Risk, 1^{st} year	Mortality Risk, 1^{st} update	Plus 1 Dose of erythropoietin
0–3	4.81	9.71	7.73	11.24
3–6	43.74	30.55	51.63	30.06
6 9	16.18	19.54	16.51	19.62
9 12	21.03	14.01	16.45	13.95
12–15	5.43	11.59	3.99	11.38
15–18	3.94	8.91	2.29	8.51
...	...	...	...	...

Table 2. Marginal posterior distribution for patient's mortality risk and its changing when decision on erythropoietin dose is made.

risk profile for a patient and main causes on horizon period; on each distribution we apply the Euclidean measure of distance to compare the profiles [Eq. 10].

The score, defined by the distance, prioritizes the variables on the basis of importance for determining the risk profile. At start of the study the patient's risk profile is better then the reference one, while since 1^{st} update the probability of mortality increases.

Variable State	Marginal posterior distribution in % Reference Profile	Haemoglobin, Hgb 1^{st} year	Haemoglobin, Hgb 1^{st} update	Decision Plus 1 Dose of erythropoietin
0 9	22.55	8.56	21.66	20.11
9–10	31.22	50.61	24.09	31.71
10–11	19.08	31.17	51.82	20.03
11–12	13.01	4.58	1.15	13.56
12 13	8.67	3.2	0.82	8.77
13 14	3.3	1.13	0.28	3.49
14–20	2.17	0.73	0.18	2.40
Variable State	Marginal posterior distribution in % Reference Profile	Serum Ferritin>100 1^{st} year	Serum Ferritin>100 1^{st} update	Decision Plus 1 Dose of erythropoietin
False	0.001	76.47	57.32	8.59
True	99.99	23.53	42.68	1.41
Variable State	Marginal posterior distribution in % Reference Profile	Erythropoietin dose 1^{st} year	Erythropoietin dose 1^{st} update	Decision Plus 1 Dose of erythropoietin
0 1000	12.64	2.97	1.54	12.9
0–6000	44.93	10.57	5.48	45.01
6000–11000	29.12	83.32	91.36	29.89
11000–16000	8.26	2.18	1.13	8.39
...	...	...	...	...

Table 3. Some marginal posterior distributions for patient's mortality risk profile and related changing for decision.

Analyzing the mortality causes and their related variables, the most important causes are mainly due to iron cycle problems (i.e. Haemoglobin, Serum Ferritin and Erythropoietin dose); there are many failures for serum ferritin and more doses of erythropoictin then the standard quantity are administered [Tab. 3].

In this example, nephrologists and risk managers have to decide if the iron management therapy (i.e. augmenting erythropoictin dose) is suitable for the patient; in Table 2 we can see that the probability of low mortality risk levels (0–3, and 3–6) increases and also the probability of the cause states which provoke mortality decrease [Tab. 3].

5 Conclusions

The national and international state of the art upon clinical and operational risks is only beginning. Organizations and their governance have understood a little bit of the importance of risk management as a key element to improve the service delivery and patient outcome. Furthermore, this question is also related to the general uncertainty about what is clinical and operational risk and how we can manage them.

The reduction of uncertainty merging the scientific literature results into a unique Bayesian framework allows to identify the most important operational and clinical risks which may generate adverse events. In particular, the identification of the mathematical relationships among medical

variables allows to understand the most important key variables for the risk profile of each patient and department. The new structured knowledge is the basis for the realization of the model for the risk assessment phase and these results are new inputs for decision making.

In the haemodialysis application, the resulting Bayesian Network is validated both from a medical and statistical point of view, by distance measures. Comparing the risk profiles among different patients or between patient's and reference's one we identify unacceptable risks and the most important causes of failure.

The complexity of the model was augmented introducing decisional and quality nodes, compliance with national and international guidelines and scientific results. The formalization of the most important decisions help risk managers and governance to make the best decision for patient, banking business lines or department, given the context of information and knowledge.

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References

1. Alexander C. *Operational Risk: Regulation, Analysis and Management*. Financial Times Prentice Hall, 2003.
2. Berger J.O. *Statistical Decision Theory and Bayesian Analysis*. Springer Verlag, New York, 1985.
3. Basel Committee on Banking Supervision. International convergence of capital measurement and capital standards. a revised framework. Technical Report 137, Basel Committee on Banking Supervision, 2004.
4. Chib S., Clyde M., Woodworth G. and Zaslavsky A. *Subjective and objective Bayesian Statistics. Principles, models, and applications*. S. James Press, Wiley, second edition, 2002.
5. Commissione Tecnica sul Rischio Clinico. Risk management in sanit. Il problema degli errori. Technical Report DM 5/3/03, Ministero della Salute, 2004.
6. Cornalba C. *Risk management models in health care: methods and applications for the assessment of clinical and operational risk*. Ph.D. Thesis, 2006.
7. Cornalba C. and Giudici P. Bayesian networks for operational risk management, compliant with Basel II. In Universit di Padova CLUEP, editor, *SIS, XLIII Riunione Scientifica, Bari (Italy)*, pages 463–466, 9-11 June 2004.
8. Giudici P. *Applied data mining*. John Wiley and Sons, 2003.
9. Jensen F.V. *Bayesian Networks and Decision Graphs*. Springer, First edition, 2001.
10. Lorton L. *Risk Assessment and Management Guide for the Medical Practice*. AMA Press, American Medical Association, 2005.
11. McCulloch C.E. and Searle S.R. *Generalized, Linear, and Mixed Models*. Wiley Series in Probability and Statistics, 2001.
12. Russell S. and Norvig P. *Artificial Intelligence: A Modern Approach*. Prentice Hall, 1995.
13. Savage L.J. *Foundations of statistics*. John Wiley and Sons, New York, 1954.
14. Wald A. *Statistical Decision Functions*. John Wiley and Sons, Inc, New York, 1950.